Heat Release Rate Calculation

1 Kawagoe

$\dot{Q} = 1500 \times A_o \times \sqrt{H_o}$

Example:
There is a fully developed fire in a room. The room has a ceiling height of 2.5 meters. The length and width are 5 and 4 meters.

The only opening to the room is an open door with a width of 1 meter and a height of 2 m.

The maximum heat release rate, calculated with Kawagoe's formula, will then be 4.24 MW:

$\dot{Q} = 1500 \times 2 \times \sqrt{2} = 4243\ kW = 4.24\ MW$

What if there are multiple openings? What if there was an additional window that breaks with a height of 1.02 m and a width of 0.76 m?

2 Simple approach

Take the weighted average of the openings

$A_{tot} = A_1 + A_2 = 1 \times 2 + 1.02 \times 0.76 = 2.78\ m^2$

$$H_{avg} = \frac{A_1 \times H_1 + A_2 \times H_2}{A_{tot}} = \frac{2 \times 2 + 0.78 \times 1.02}{2.78} = 1.73\ m$$

$\dot{Q} = 1500 \times 2.78 \times \sqrt{1.73} = 5469\ kW = 5.47\ MW$

3 Complex approach

3.1 For one opening

Let's look at Kawagoe's formula in detail:

An assumption needs to be made about the gas temperature inside the enclosure. Let's assume the gas temperature is 1000 °C. Let's also assume that it is uniform across the room. We also assume that the ambient temperature is 20 °C.

When the door is the only opening, two flows establish through the door: a mass flow of (burning) gases towards the outside and a mass flow of fresh air towards the inside.

We can calculate these flows with the following equation:

$$\dot{m} = C_d \int_A \rho \times v \times dA$$

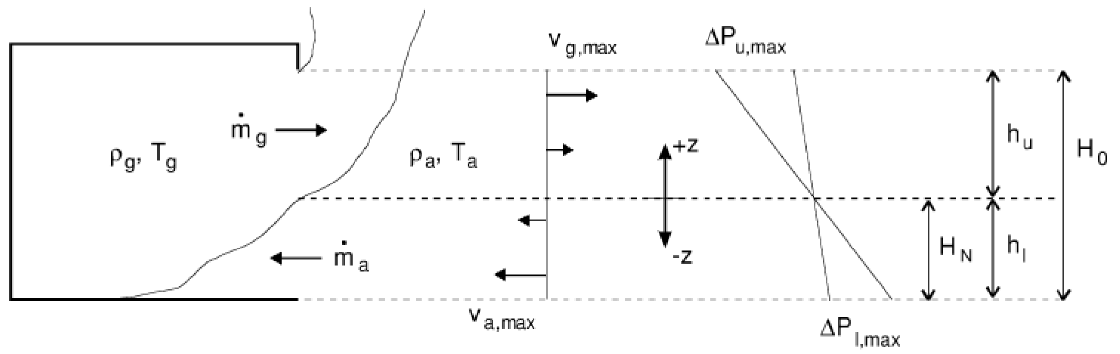

FIGURE 5.12 The well-mixed case: An enclosure with uniform temperature, T_g, higher than the outside temperature, T_a.

Figure 1 Extract from [2]: velocity and pressure profiles as well as the denotations used

In the figure above, the reference of the calculations is the neutral plane. The velocity increases as a function of z. The higher above the neutral plane, the higher the velocity. The lower below the neutral plane, the higher the velocity.

However, we assume a constant velocity in any horizontal plane. Therefor:

$$\dot{m} = C_d \int_0^z \rho \times W \times v_z \times dz$$

Velocity above the neutral plane can be written as:

$$v_g(z) = \sqrt{\frac{2 \times z \times (\rho_a - \rho_g) \times g}{\rho_g}}$$

And below the neutral plane as:

$$v_a(z) = \sqrt{\frac{2 \times z \times (\rho_a - \rho_g) \times g}{\rho_a}}$$

Now we can calculate the mass flow of hot gasses as such:

$$\dot{m}_g = C_d \times W \times \rho_g \int_0^{h_u} \sqrt{\frac{2 \times z \times (\rho_a - \rho_g) \times g}{\rho_g}} dz$$

$$\dot{m}_g = C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \int_0^{h_u} \sqrt{z} \times dz$$

$$\dot{m}_g = \frac{2}{3} \times C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \times {h_u}^{3/2}$$

In a similar way we can calculate the mass flow of fresh air:

$$\dot{m}_a = \frac{2}{3} \times C_d \times W \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times {h_l}^{3/2}$$

Both equations use a distance that refers to the opening:

- h_u between the neutral plane and the top of the opening
- h_l between the neutral plane and the bottom of the opening

However, the exact location of the neutral plane is not known. We do know that what flows in must flow out due to the law of conservation of mass.

$$\dot{m}_g = \dot{m}_a$$

$$\frac{2}{3} \times C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \times {h_u}^{3/2} = \frac{2}{3} \times C_d \times W \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times {h_l}^{3/2}$$

$$\rho_g \times \sqrt{\frac{1}{\rho_g}} \times {h_u}^{3/2} = \rho_a \times \sqrt{\frac{1}{\rho_a}} \times {h_l}^{3/2}$$

$$\sqrt{\rho_g} \times {h_u}^{3/2} = \sqrt{\rho_a} \times {h_l}^{3/2}$$

$$\frac{{h_u}^{3/2}}{{h_l}^{3/2}} = \frac{\sqrt{\rho_a}}{\sqrt{\rho_g}}$$

$$\left(\frac{h_u}{h_l}\right)^{3/2} = \left(\frac{\rho_a}{\rho_g}\right)^{1/2}$$

$$\frac{h_u}{h_l} = \left(\frac{\rho_a}{\rho_g}\right)^{1/3}$$

$$H_o = h_u + h_l$$

$$\frac{H_o - h_l}{h_l} = \left(\frac{\rho_a}{\rho_g}\right)^{1/3}$$

$$h_l = \frac{H_o}{1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}}$$

$$\dot{m}_a = \frac{2}{3} \times C_d \times W \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times \left(\frac{H_o}{1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}}\right)^{3/2}$$

$$\dot{m}_a = \frac{2}{3} \times C_d \times W \times H_o \times \sqrt{H_o} \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times \left(\frac{1}{1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}} \right)^{3/2}$$

$$\dot{m}_a = \frac{2}{3} \times C_d \times A_o \times \sqrt{H_o} \times \rho_a \times \sqrt{2 \times g} \sqrt{\frac{(\rho_a - \rho_g)}{\rho_a}} \times \left(\frac{1}{\left[1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}\right]^3} \right)^{1/2}$$

$$\dot{m}_a = \frac{2}{3} \times C_d \times A_o \times \sqrt{H_o} \times \rho_a \times \sqrt{2 \times g} \times \sqrt{\frac{\frac{(\rho_a - \rho_g)}{\rho_a}}{\left[1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}\right]^3}}$$

The last part of the equation is called the density factor: $\sqrt{\frac{\frac{(\rho_a - \rho_g)}{\rho_a}}{\left[1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}\right]^3}}$. This factor is only dependent on the ambient temperature and the gas (smoke) temperature. It is possible to evaluate how the density factor evolves when the temperature in the enclosure increases. In a graph, it looks like this:

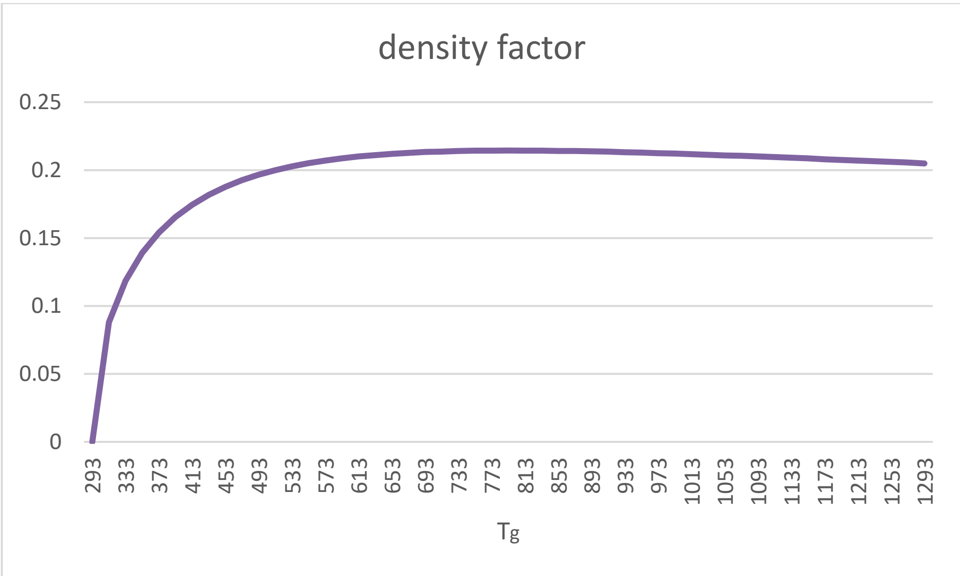

Figure 2 The evolution of the density factor as a function of gas temperature. The density factor remains more or less constant in a fully developed fire.

The highest value of the density factor is 0.214. This is reached when the gas temperature is between 713 K and 893 K or 440 °C and 620 °C. In the temperature field between 800 °C and 1000 °C, the value of the density factor ranges from 0.210 and 0.206. Nevertheless, in order to achieve a conservative result, the calculations are continued with 0.214, the maximum value that is possible.

The equation now looks like this:

$$\dot{m}_a = \frac{2}{3} \times C_d \times A_o \times \sqrt{H_o} \times \rho_a \times \sqrt{2 \times g} \times 0.214$$

We use the following values for the constants
$C_d = 0.7$
$g = 9.81$
$\rho_a = 1.2$

When these constants are put in the equation, the constants evolve to 0.53. The equation then looks like:

$$\dot{m}_a = 0.53 \times A_o \times \sqrt{H_o}$$

If the 0.214 is replaced by 0.206, corresponding to 980 °C, the constant becomes 0.51. It becomes 0.5 at a temperature of 1160 °C.

The value of 0.5 is selected to continue and the relation to calculate the mass flow becomes: $\dot{m}_a = 0.5 \times A_o \times \sqrt{H_o}$

The above equation expresses the amount of air that flows into the enclosure. We know that the heat of combustion of air is 3 MJ/kg. Therefore:

$$\dot{Q} = 1500 \times A_o \times \sqrt{H_o}$$

3.2 For two openings

What if there was an additional window that breaks with a height of 1.02 m and a width of 0.76 m? Let's assume that the top of the window is at the same height as the top of the door. This means that the window sill is at a height of 0.98 m.

Let's have a look at the height of the neutral plane in the approach of a single opening.

$$h_l = \frac{H_o}{1 + \left(\frac{\rho_a}{\rho_g}\right)^{1/3}}$$

In our case, the height of the door was 2 m. We assumed 1000 °C inside and 20 °C outside. This corresponds with a density of 0.277 kg/m³ inside and 1.2 kg/m³ outside.

$$h_l = \frac{2}{1 + \left(\frac{1.2}{0.277}\right)^{1/3}} = 0.76\ m$$

Let's assume that the opening of the window does not lead to a neutral plane higher than 0.98 m.

In this case the window becomes a unidirectional outlet. The mass flow through the window can be calculated as:

$$\dot{m}_{g,w} = C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \int_{b_w - h_l}^{t_w - h_l} \sqrt{z} \times dz$$

With:

- t_w: the top of the window opening
- b_w: the window sill

$$\dot{m}_{g,w} = C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \left[\frac{2}{3}(t_w - h_l)^{3/2} - \frac{2}{3}(b_w - h_l)^{3/2}\right]$$

The mass flow of smoke through the door can be calculated with the same expression as before:

$$\dot{m}_{g,d} = \frac{2}{3} \times C_d \times W \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \times {h_u}^{3/2}$$

The same goes for the mass flow of fresh air through the bottom part of the door:

$$\dot{m}_a = \frac{2}{3} \times C_d \times W \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times {h_l}^{3/2}$$

The conservation of mass still applies, so:

$$\dot{m}_{g,d} + \dot{m}_{g,w} = \dot{m}_a$$

$$\frac{2}{3} \times C_d \times W_d \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \times {h_u}^{3/2} + C_d \times W_w \times \rho_g \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_g}} \left[\frac{2}{3}(t_w - h_l)^{3/2} - \frac{2}{3}(b_w - h_l)^{3/2}\right] = \frac{2}{3} \times C_d \times W_d \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times {h_l}^{3/2}$$

We can now simplify the equation:

- By eliminating 2/3 at both sides
- By assuming that C_d is the same for both openings.
- By eliminating $\sqrt{2 \times (\rho_a - \rho_g) \times g}$ at both sides

$$W_d \times \rho_g \times \sqrt{\frac{1}{\rho_g}} \times {h_u}^{3/2} + W_w \times \rho_g \times \sqrt{\frac{1}{\rho_g}} \left[(t_w - h_l)^{3/2} - (b_w - h_l)^{3/2}\right] = W_d \times \rho_a \times \sqrt{\frac{1}{\rho_a}} \times {h_l}^{3/2}$$

$$W_d \times \sqrt{\rho_g} \times {h_u}^{3/2} + W_w \times \sqrt{\rho_g}\left[(t_w - h_l)^{3/2} - (b_w - h_l)^{3/2}\right] = W_d \times \sqrt{\rho_a} \times {h_l}^{3/2}$$

$H_d = h_u + h_l$

$h_u = H_d - h_l$

$$W_d \times \sqrt{\rho_g} \times (H_d - h_l)^{3/2} + W_w \times \sqrt{\rho_g}\left[(t_w - h_l)^{3/2} - (b_w - h_l)^{3/2}\right] = W_d \times \sqrt{\rho_a} \times {h_l}^{3/2}$$

This is a more complex equation but it can be solved numerically (by “trial and error”).

The numerical solution for h_l is 0.94 m.

Let’s have a look at the mass flow of fresh air through the door opening:

$$\dot{m}_a = \frac{2}{3} \times C_d \times W_d \times \rho_a \times \sqrt{\frac{2 \times (\rho_a - \rho_g) \times g}{\rho_a}} \times {h_l}^{3/2}$$

$$\dot{m}_a = \frac{2}{3} \times C_d \times W_d \times \sqrt{\rho_a\left(2 \times (\rho_a - \rho_g) \times g\right)} \times {h_l}^{3/2}$$

$\dot{m}_a = 2.091\ kg/s$

When 3 MJ/kg is used as a heat of combustion for air, the calculated heat release rate becomes 6.272 MW. This is 15% more than calculated with the simple approach.

Compared to the single opening, a simplified approach does not seem possible.
The height of the neutral plane is dependent on the size of the openings and dependent on the chosen gas temperature.

4 References

[1] *Drysdale D (1999) Introduction to fire dynamics, 2nd edition, chapter 10*
[2] *Karlson B, Quintiere J (2000) Enclosure fire dynamics, chapter 5*